Harvard-Smithsonian Center for Astrophysics

Precision Astronomy Group

To:	Distribution	June 14, 2001	TM01-04
From:	J.D. Phillips		
Subject:	Blur due to optical distortion in FAME		

This memo calculates the blurring of images due to optical distortion in FAME. The reduction in precision is of the order of 5%, small but not negligible. Not considered here is the effect on image shape [see Zacharias, Memo 13], which might present an unacceptable computational burden, or result in systematic error if adequate computational resources cannot be brought to bear.

The ideal mapping from sky to focal plane is

$$u = f\theta_u \tag{1}$$

where u is position in the focal plane in the scan direction, f is the focal length and θ_u is angle measured about the spacecraft rotation axis. Even in an optical system that is as free of distortion as possible, there is a type of distortion due to the projection of the celestial sphere onto the planar detector surface, discussed in Appendix A. For FAME, this projection distortion is small. The more significant effect of optical distortion is calculated below.

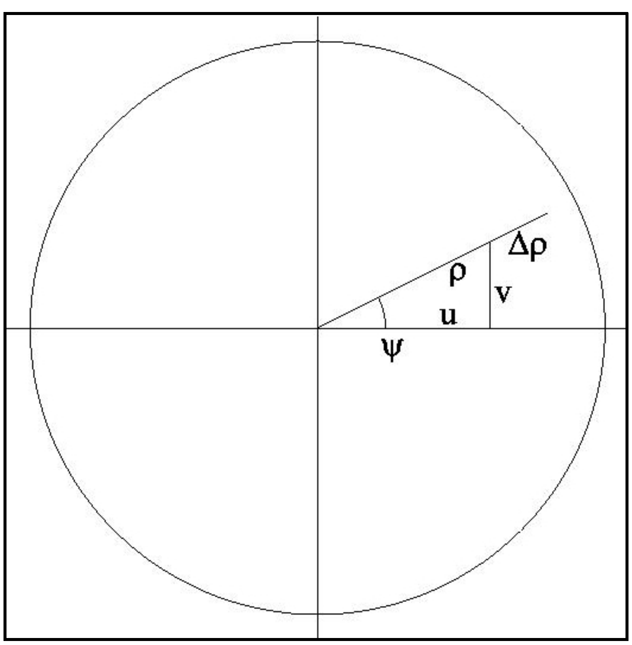

Figure 1. Image point (u,v) in FOV.

"Optical distortion" refers to a particular Seidel aberration, in which the effective focal length varies as a function of radius in the field of view (FOV), resulting in a radial perturbation of image position that is cubic in radius. Distortion leaves the image shape at a given point on the focal plane unchanged, to excellent approximation. However, variations of image speed during the integration across the CCD can give rise to changes of shape, which will require modeling in the data reduction. Other aberrations such as coma also change the image shape. The shape changes are significant, but are not treated here.

Let v represent position in the focal plane in the cross-scan direction (Fig. 1),

$\rho = \sqrt{u^2 + v^2}$ be the radius of a point from the center of the FOV, and ψ be its polar angle. Distortion shifts the image by a radial distance

$$\Delta\rho = \alpha\rho^3 \ . \qquad \textbf{(2)}$$

For FAME without the aspheric distortion corrector, $\alpha = 0.0135$ m/m^3, or 45 μm at the edge of the FOV. In components, the star shift is

$$\begin{aligned}\Delta u &= \Delta\rho\cos\psi \\ \Delta v &= \Delta\rho\sin\psi \ .\end{aligned} \qquad \textbf{(3)}$$

Table 1. Quantities used.

Quantity	**Sym.**	**Value**
Focal length	f	15017.26 mm
Distortion coefficient	α	0.0135 m/m^3 (w/o aspheric corrector)
Pixel size	w	15 μm
Number of pixels (scan) (cross-scan)	 N_S N_C	 4096 2048
FOV radius (linear)	ρ_o	0.149 m
FOV radius (angular)	ρ_o/f	0.570000 deg

Noting that $\sin\psi = v/\rho$ and $\cos\psi = u/\rho$, and combining (2) and (3), we obtain

$$\begin{aligned}\Delta u &= \alpha\, u\,(u^2 + v^2) \\ \Delta v &= \alpha\, v\,(u^2 + v^2) \ .\end{aligned} \qquad \textbf{(4)}$$

Along the axes (u=0 or v=0) the distortion in the non-zero coordinate is cubic, but on a circle centered on the center of the FOV it is linear. This gives shape to Fig. 3 of [Horner, FAME EM-0005], in which the distortion Δu is plotted as height over the uv plane. The edge of the FOV is circular. The distortion on this edge appears in EM-0005 as an ellipse lying in a tilted plane.

Cross-scan

To compensate for cross-scan image motion, the CCD can be rotated to best align with the local direction. Even without the aspheric distortion correction, the needed rotation is less than one pixel out of 4096. The shift will not be treated herein.

In-scan

The in-scan component Δu shifts and blurs the image. The shift is up to ~3 pixels. The resulting bias in the astrometric data can be removed in the analysis by estimating the parameters of a model that treats the cubic term, and probably other terms as well, such as those due to manufacturing and alignment errors. An accuracy of better than one part in 10^3 is required on the cubic term, but it will be a slowly-varying function of time.

The variation of the in-scan component Δu represents a varying image velocity, which blurs the detected image, so degrades the photon-limited precision, σ.

The mean image position is shifted by $\langle\Delta u\rangle$, where the average is taken over one CCD column. The RMS blur is then

$$b = \sqrt{<(\Delta u - <\Delta u>)^2>} = \sqrt{<\Delta u^2> - <\Delta u>^2} \ . \qquad \textbf{(5)}$$

The velocity with which the CCD's are clocked will be adjusted to minimize the blur, considering all CCD's[1]. (The astrometric chips are the primary concern.) The in-scan distortion with adjusted clock is

$$\Delta u = \alpha \left[u \left(u^2 + v^2 \right) - \beta \rho_o^2 u \right] , \qquad \textbf{(6)}$$

where β is the clock rate adjust parameter, and ρ_o is the (linear) radius of the FOV. For β=1, the clock rate adjustment cancels the image displacement due to distortion at the edge of the FOV. One may take the mean square of (6), integrate over the whole FOV, and minimize with respect to β. This yields β = 3/5. However, an accurate measure of the blur can only be obtained by integrating over just the CCD column in question. Basing the choice of β on such an integration may yield improved mission precision. Below, I give the integration over a column, and present results for one column of each CCD.

The RMS blur in a CCD column is obtained by putting (6) into (5), and taking the averages from (u_1,v) to (u_2,v). We get

$$b = \alpha \left\{ \frac{1}{u_2 - u_1} \left[\frac{1}{7}(u_2^7 - u_1^7) + \frac{2}{5}(v^2 - \beta\rho_o^2)(u_2^5 - u_1^5) + \frac{1}{3}(v^2 - \beta\rho_o^2)^2 (u_2^3 - u_1^3) \right] - \frac{1}{(u_2 - u_1)^2} \left[\frac{1}{4}(u_2^4 - u_1^4) + \frac{1}{2}(v^2 - \beta\rho_o^2)(u_2^2 - u_1^2) \right]^2 \right\}^{1/2} . \qquad \textbf{(7)}$$

This result does not simplify nicely, but it allows convenient numerical evaluation of the blur, and optimization with respect to β.

With values as given in Table 1, the blur for each CCD is given in Table 2, for two values of the clock rate adjust parameter, β. The CCD positions used were taken from Horner, EM-0005, and are plotted directly from that numerical data in Fig. 2. The blur was calculated for a column centered in cross-scan. Two choices of β are shown in the table, β=0.6, which is the value that minimizes an RMS of blur over the whole focal plane, and is close to the value that minimizes the RSS of RMS values across each of the FAME astrometric chips. With this value of β, the two astrometric chips closest to the edge of the FOV, 11 and 14, see an RMS blur of 5.3 μm, and the rest have blur of 2.0 μm or under. The other value, β=1.0, reduces the maximum blur, but 10 astrometric chips have blur from 3.3 to 4.0 μm, and the RSS is increased.

One measure of the way blur affects mission accuracy is the RMS of the RMS values for each chip alone; this is given in the bottom row of the table. This measure is clearly wrong, in the limit in which a few chips have blur so extreme that they don't contribute usefully to mission information. The blur from these chips, however, dominates the RMS of the RMS blur values. Nonetheless, in the limit that the blur from distortion is small compared with the width of the

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[1] Although adjusting each CCD's clock rate separately would reduce the blur due to distortion, for simplicity of the onboard electronics, all FAME CCD's will clock at the same rate.

PSF, the RMS of the RMS may be a useful measure. The correct measure is mission accuracy, which one can calculate if the effect of blur on photon-noise-limited error is known.

To optimize β based on mission accuracy, one would start with an expression giving an estimate of $\sigma(\theta)$, as can be obtained from a covariance study of the centering of images blurred by various values θ. The statistical portion of mission accuracy is

$$\sigma_m = \left\{ \sum_i \frac{1}{\sigma(\theta_i)^2} \right\}^{-1/2} \quad \textbf{(8)}$$

where i runs over all observations, and θ_i is the rms blur for the i-th observation. One then wishes to choose β so as to minimize σ_m. Assuming that all CCD's are visited roughly equally by all stars[2], one may simplify the evaluation of (8) by making the sum run over all CCD's, with θ_i the rms blur in the i-th CCD. Such an optimization may be the subject of a future memo.

Estimate of the effect on precision.

An estimate of the effect of the blur shown in Table 2 can be obtained by estimating the resultant broadening of the PSF. For two Gaussian functions, the RMS of their convolution is the RSS of the individual RMS's. For a variety of other functional forms, this turns out to be approximately true. The $\mathrm{sinc}^2(x)$ PSF for the rectangular aperture does not have a finite RMS value, so I take the RSS of the FWHM of the PSF and of the blur. To obtain a Gaussian function representing the blur, I use the RMS over astrometric CCD's mentioned above, with β=0.6. The RMS is 2.2 µm, and the FWHM of such a Gaussian is 5.2 µm. The FWHM of the PSF for a 6000 K star is 15.9 µm. Taking the RSS of these yields an estimate for the net FWHM of 16.7 µm. This is an increase of only 5%. The photon-limited mission precision may be expected to worsen by a similar percentage.

[2] For example, all faint-star CCD's visited by all faint stars.

Table 2. Blur for each CCD, for two choices of β. u_1, u_2, and v are in units of the FOV radius ρ_o. Photometric chips have a 0 in column "Ast," and their blur values are written in strikeout text. The rows for the two astrometric chips with greatest blur (for β=0.6) are shaded gray. The data for CCD positions in columns u_1, u_2, and v are from [Horner, EM-0005]. The row marked "RMS" is the RMS of the blur for the astrometric chips.

CCD	Ast	u_1	u_2	v	RMS blur, µm	
					β = 0.6	β = 1.0
1	1	-0.2056	0.2056	-0.8718	0.9227	1.0668
2	1	-0.4503	-0.0390	-0.6539	0.4229	1.8664
3	1	0.0377	0.4489	-0.6539	0.4174	1.8755
4	1	-0.6936	-0.2823	-0.4360	1.8112	0.8489
5	1	-0.2056	0.2056	-0.4360	1.9107	3.8967
6	1	0.2823	0.6936	-0.4360	1.8112	0.8489
7	0	-0.9382	-0.5270	-0.2179	~~5.5022~~	~~3.5849~~
8	1	-0.4503	-0.0390	-0.2179	1.7698	3.7331
9	1	0.0377	0.4489	-0.2179	1.7788	3.7425
10	0	0.5256	0.9368	-0.2179	~~5.4717~~	~~3.5552~~
11	1	-0.9382	-0.5270	0.0000	5.2718	3.3626
12	1	-0.4503	-0.0390	0.0000	2.0005	3.9677
13	1	0.0377	0.4489	0.0000	2.0097	3.9772
14	1	0.5256	0.9368	0.0000	5.2414	3.3331
15	0	-0.9382	-0.5270	0.2179	~~5.5022~~	~~3.5849~~
16	1	-0.4503	-0.0390	0.2179	1.7698	3.7331
17	1	0.0377	0.4489	0.2179	1.7788	3.7425
18	0	0.5256	0.9368	0.2179	~~5.4717~~	~~3.5552~~
19	1	-0.6936	-0.2823	0.4359	1.8109	0.8490
20	1	-0.2056	0.2056	0.4359	1.9109	3.8970
21	1	0.2823	0.6936	0.4359	1.8109	0.8490
22	1	-0.4503	-0.0390	0.6539	0.4229	1.8664
23	1	0.0377	0.4489	0.6539	0.4174	1.8755
24	1	-0.2056	0.2056	0.8718	0.9227	1.0668
RMS					2.2218	2.8231

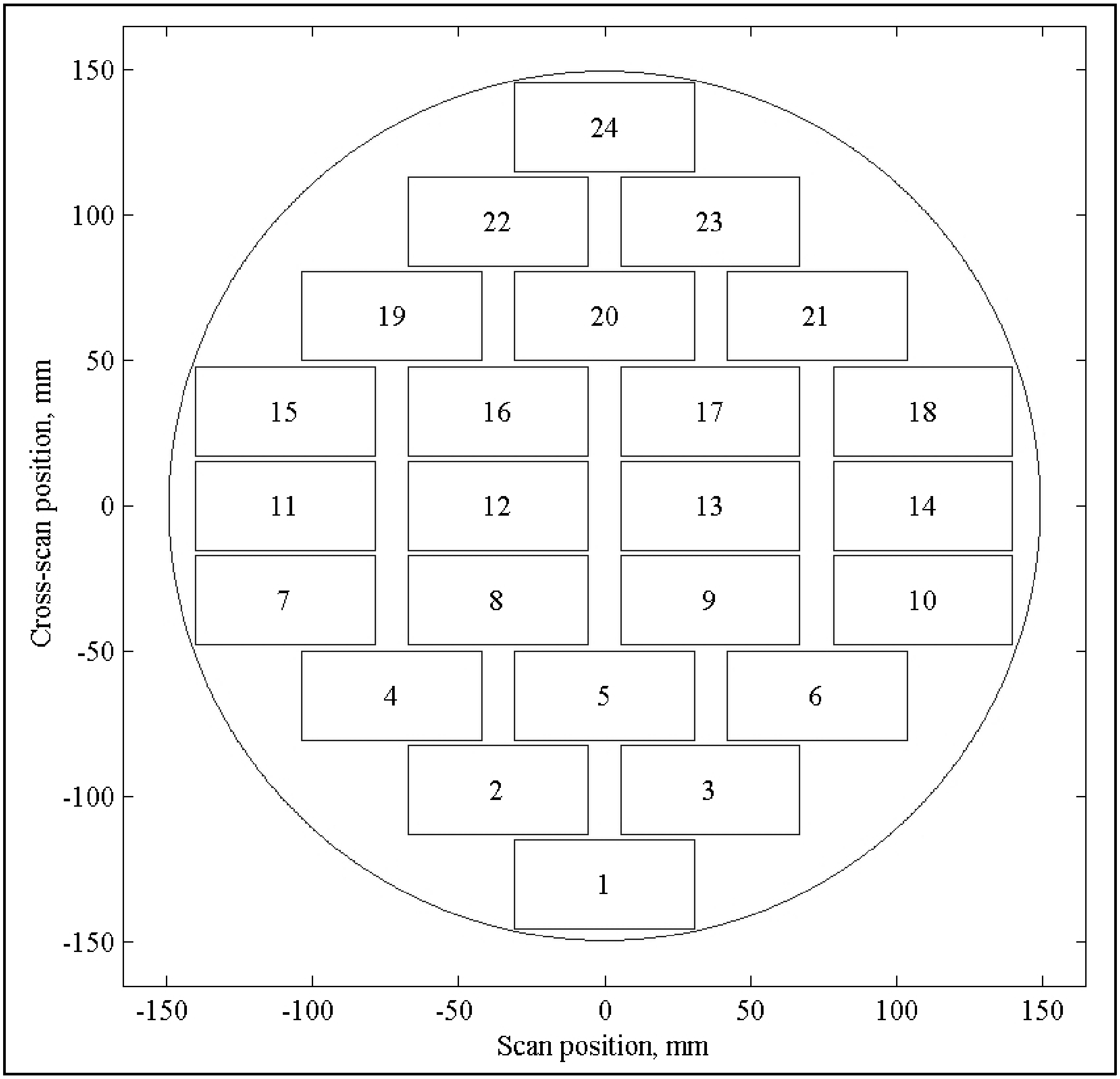

Figure 2. CCD positions, plotted from the data given in Table 2, and used in calculating blur. Circle represents edge of FOV as defined in Table 1.

References

Horner, S., "Optical distortion and the associated image smear.", Lockheed-Martin Engineering Memo 0005, 12 Feb., 2001

Zacharias, N., "Memo 13: PSF's with 3rd order optical distortion applied," e-mail to FAME PSF discussion group, 2/13/01.

Appendix A. Projection from the celestial sphere to the focal plane.

This appendix treats a small, irreducible distortion arising from the mapping of a sphere onto a plane.

Consider a star at point C (Fig. 3). The center of one of the telescope fields of view is at A. Arcs AB and AC have length θ_u and θ_v, respectively. (All arcs named in the text of this Appendix are arcs of great circles unless otherwise specified.) The star maps to a point C′ in the focal plane. The spacecraft rotates about OP, and from the spacecraft frame, point C moves on a circle centered at Q. Arc DF (not part of a great circle) is part of this circle. When the spacecraft rotates uniformly, θ_u increases linearly with time.

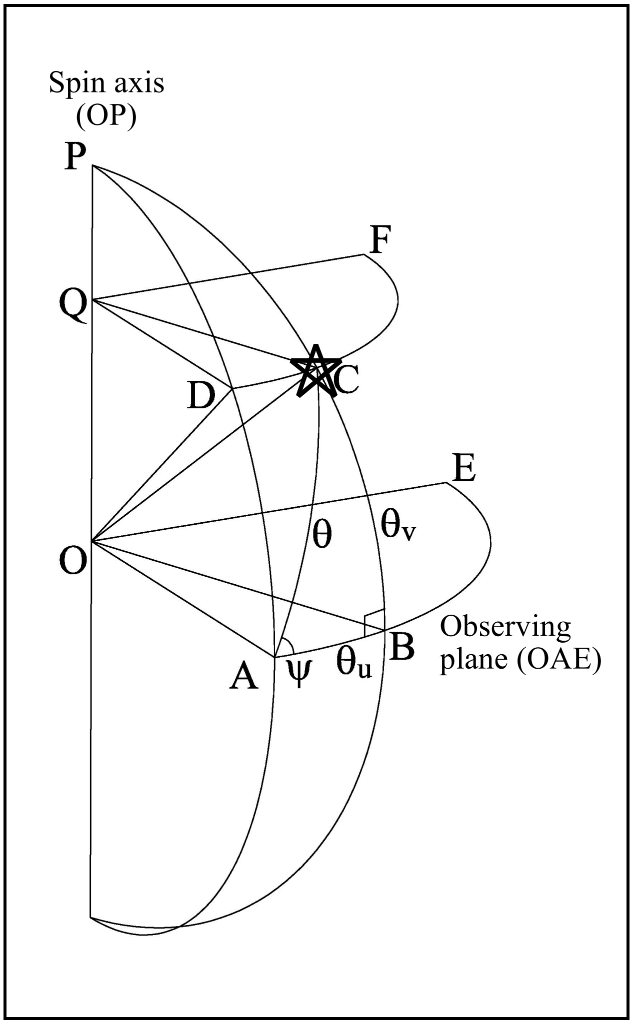

Figure 3. Geometry of the mapping.

The optical system maps points along arc AC, which makes an angle ψ with the observing plane (OAE), onto points on a line A′C′ in the focal plane. (A′C′ makes an angle ψ with the scan direction.) A system free of "f-θ" distortion has a linear relationship between the angle θ and the distance A′C′. (The constant of proportionality is called the focal length.) Optical distortion – the cubic term considered in this memo, for example -- introduces nonlinearity into the mapping of angles θ into distances A′C′.

Even in a system free of f-θ distortion, there is a small distortion due to the mapping of the celestial sphere onto the focal plane. This distortion is small enough to ignore at the present stage of FAME's design, but is calculated here for reference. It shifts images by a distance of the order of 1 μm, i.e., 14 milliarcsec.

We need to calculate the length of arc AC, θ, and the angle ψ. Consider the spherical triangle ABC. Angle ABC is 90°. We need the analog of Pythagoras' Theorem for spherical triangles,

$$\cos\theta = \cos\theta_u \cos\theta_v \ . \qquad \textbf{(9)}$$

We need another relation for right spherical triangles,

$$\cos\psi = \tan\theta_u \cot\theta. \tag{10}$$

This allows us to compute θ and ψ. For an axisymmetric optical system, ψ is the same in FOV and focal plane. In the focal plane, (ρ,ψ) form the (2-D) polar coordinates of the image point, with ψ measured from the u-axis. The distortion is characterized by ρ(θ). For this memo, I take the optical system to be axisymmetric, neglecting FAME's off-center aperture. For this appendix, I assume zero f-θ distortion. (In the body of the memo, I calculated the blurring that would result from the distortion of FAME's actual optical system prior to the addition of the aspheric corrector.) The coordinates of the image point are

$$\begin{aligned} u &= \rho\cos\psi \\ v &= \rho\sin\psi \ . \end{aligned} \tag{11}$$

We may take the u motion thus calculated, and subtract $f\theta_u$, which is the value that would result from image motion at a constant velocity that is independent of θ_v.

While it is easy to substitute (9) into (10) to obtain expressions for θ and ψ in terms of θ_u and θ_v, it would involve some effort to expand the resulting expression to find the terms of leading order in θ_u and θ_v. Therefore, I have simply evaluated (9) through (11) numerically. The results are shown in Fig. 4. While the curves for each value of θ_v are very nearly straight lines, the different slopes imply that the images will traverse the focal plane at different speeds. The FAME CCD’s will all be clocked at the same rate, so the different speeds give rise to blur, but not much of it: the peak to peak displacement in Fig. 4 is 2 µm, and the rms is even smaller.

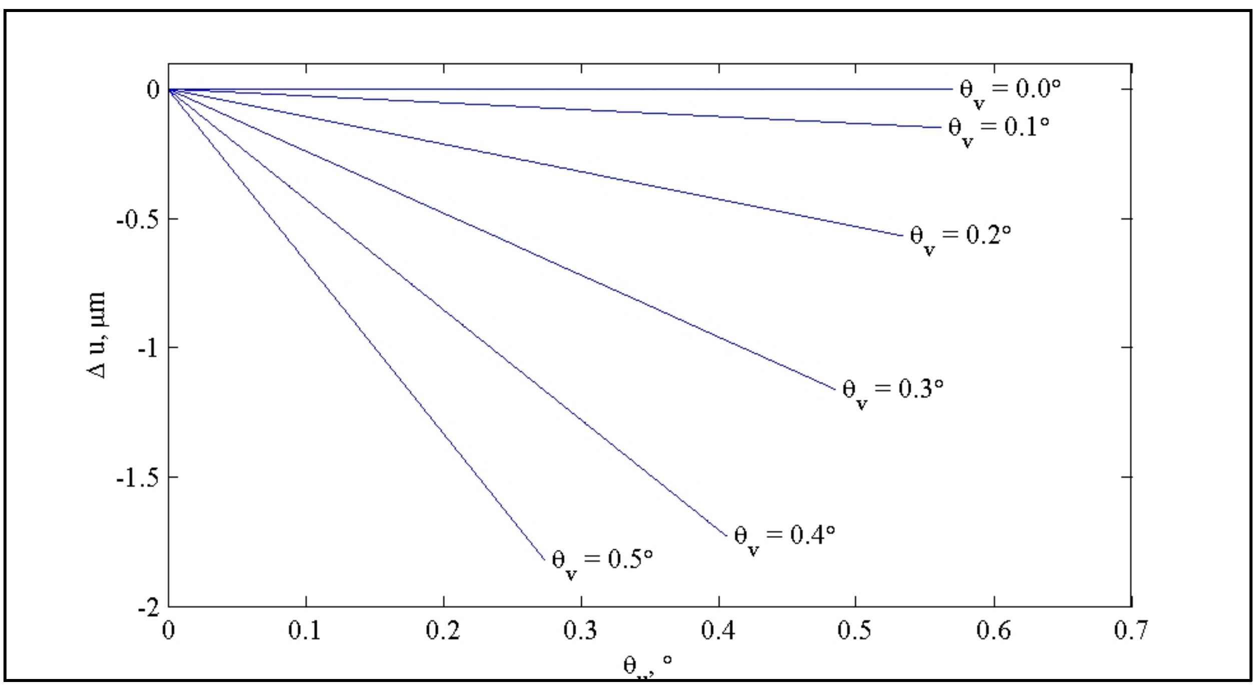

Figure 4. Image motion due to projection distortion, for several cross-scan angles θ_v. Motion with zero f-θ distortion, using the nominal focal length f, has been subtracted. The individual curves are straight lines, to within about 1 part in 10^5. For each value of θ_v, the curve terminates at the edge of a focal plane of 0.57° radius. Slopes of the lines are given by -27

An alternate approach to the spherical triangles used above, perhaps more familiar to the modern reader, uses vectors. Let the x-axis be along OA, y be along OE, and z be along OP. Then taking the sphere, of which a piece is shown, to have unit radius, point A is at (1,0,0), and C is at $(\cos\theta_u \cos\theta_v,\ \sin\theta_u \cos\theta_v,\ \sin\theta_v)$. We then have that

$$\theta = \operatorname{acos}(\vec{A}\cdot\vec{C}) = \operatorname{acos}(\cos\theta_u \cos\theta_v) \qquad \textbf{(12)}$$

as before. To calculate ψ, note that it is the dihedral angle between planes OAC and OAE. Its cosine, then, is just the dot product of normals to OAC and OAE. The normal to plane OAC is

$$\hat{n}_1 = \frac{\vec{A}\times\vec{C}}{|\vec{A}\times\vec{C}|} \qquad \textbf{(13)}$$

The normal to plane OAE is $\hat{z}$. The dot product is the z component of $\hat{n}_1$. Doing the symbolic manipulations by machine,

$$\cos\psi = \frac{\sin\theta_u \cos\theta_v}{\left(1 - \cos^2\theta_u \cos^2\theta_v\right)^{1/2}} \qquad \textbf{(14)}$$

which is equivalent to (10).

Distribution

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